# Slow Extraction from the 3 TeV Injector for the VLHC

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It has been proposed to build a 3 TeV injector for the VLHC on or near the Fermilab site. Such a machine could provide fixed target beams for physics both before and during the VLHC era. This note suggests possible parameters of the extraction system.

I have chosen to use 1/2 integer extraction with 0th harmonic octupoles. This scheme is used in the Tevatron. I chose it simply because I have the formulas handy. It may not be the best scheme. I chose the septum gap to be 6 mm and to assume that both the septum and the lambertson are placed at high beta locations ($\beta$=1200 m). The septum wires are placed at $x_1$=12 mm from the center of the aperture and the outside edge is therefore at $x_2$=12+6=18 mm. Assuming an aperture of 10 mm at $\beta_{max}$ of a normal cell ($\beta_{max}$ =300 m), the septum uses about 1/3 of the available aperture. This choice appears to allow sufficient space to develop the necessary step size as the septum is approached.

The VLHC injector parameters are pretty much up in the air. I have assumed the following:

Table I

| | | |
|---|---|---|
| Beam emittance (100%) | 10 $\pi$ | mm-mrad |
| Extraction Energy | 3000 | GeV |
| Beta at septum ($\beta_s$) | 1200 | m |
| Beta at lambertson ($\beta_L$) | 1200 | m |
| Aperture at $\beta$=300m | 10 | mm |
| Septum gap | 6 | mm |
| $x_1$ | 12 | mm |
| $x_2$ | 18 | mm |

It can be shown that the extracted beam follows trajectories that circles in phase space. A possible trajectory for the extracted beam is shown in Figure 1. The radius of the circle describing the trajectory is

$$r_0^2 = -\frac{16\pi\Delta\nu}{3E}$$

where $\Delta\nu$ is the difference between the unperturbed tune and the nearest 1/2 integer (sometimes called the "tune defect") and where E is the strength of the 0th harmonic octupoles

$$E = \frac{\beta_s}{|B\rho|}\oint_C \left[\frac{\beta(z)}{\beta_s}\right]^2 \frac{1}{6}\frac{d^3B}{dx^3}\cos[2\mu(z)]dz$$

The quantity $k$ is defined as

$$k = \frac{q}{4\pi\Delta\nu}$$

where q is defined as the quadrupole fields that drive the 1/2 integer resonance.

$$q = \frac{1}{|B\rho|}\int_C \beta(z)\frac{dB}{dx}\cos[2\mu(z)]dz$$

It can be shown that the stable area is given by

$$\varepsilon_s = \frac{2\pi r_0^2}{\beta_s}\Gamma(k)$$

where

$$\Gamma(k) = \frac{2}{\pi}\left(\frac{1}{k}\sin^{-1}\sqrt{1-k} - \sqrt{k-k^2}\right)$$

and of course the stable area must be equal to the beam area to initiate the slow spill. The step size, the change in *x* per turn is given by:

$$\left.\frac{dx}{dn}\right|_{x=x_1} = 2\pi\Delta\nu p_1\left(-\frac{p_1^2 + x_1^2}{r_0^2} - k + 1\right)$$

where

$$p_1 = \sqrt{r_0^2 - \left(x_1^2 - x_c^2\right)}$$

and

$$x_c^2 = \frac{4q}{3|E|}.$$

It is desirable to have *dx/dn* large in order to obtain high efficiency, but it must be less than 1/2 the septum gap so that particles are not lost on the outside of the septum.

The following are the calculated results

Table II

| | | |
|---|---|---|
| $k$ | 0.840 | |
| $\Delta\nu$ | 0.022 | |
| $r_0$ | 11.48 | mm |
| $x_c$ | 10.52 | mm |
| 2*dx/dn* at $x_1$=12 mm | 6 | mm |
| $q$ | 3.85 | T |
| $E$ | $3.71\text{x}10^5$ | $\text{T/m}^{-2}$ |

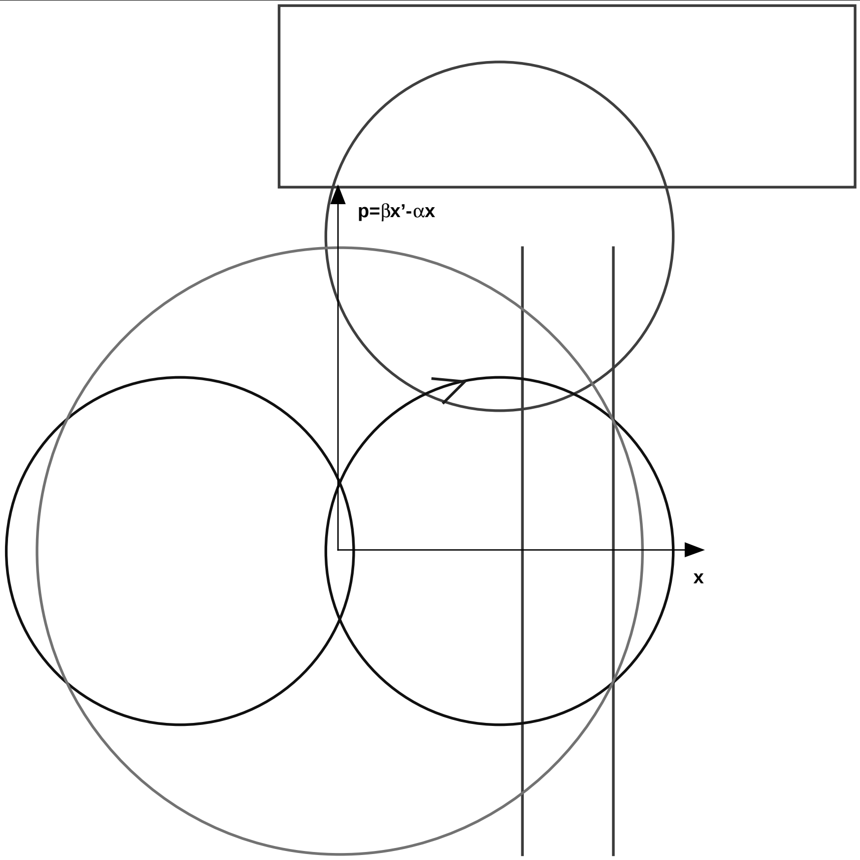


**Figure 1. Schematic of the extraction scheme.**

The strength of the quadrupoles required to drive the 1/2 integer resonance is very small. However, the octupole strength is substantial: equivalent to about 75 Tevatron spools. The requirements on the octupole strength could be reduced by chosing a different trajectory (with less curvature), but it doesn't seem like it is much of a problem to provide the octupole strength specified in Table II.

The step size as a function of *x* is shown in Figure 2. The density is equal to inverse of the step size and is shown in Figure 3. The density is approximately uniform over the septum gap and the extraction inefficiency is approximately the effective wire width divided by the septum gap. Assuming the wire width (25 μm) plus allignment tolerances lead to an effective width of 60 μm, the extraction efficiency would be 99%.

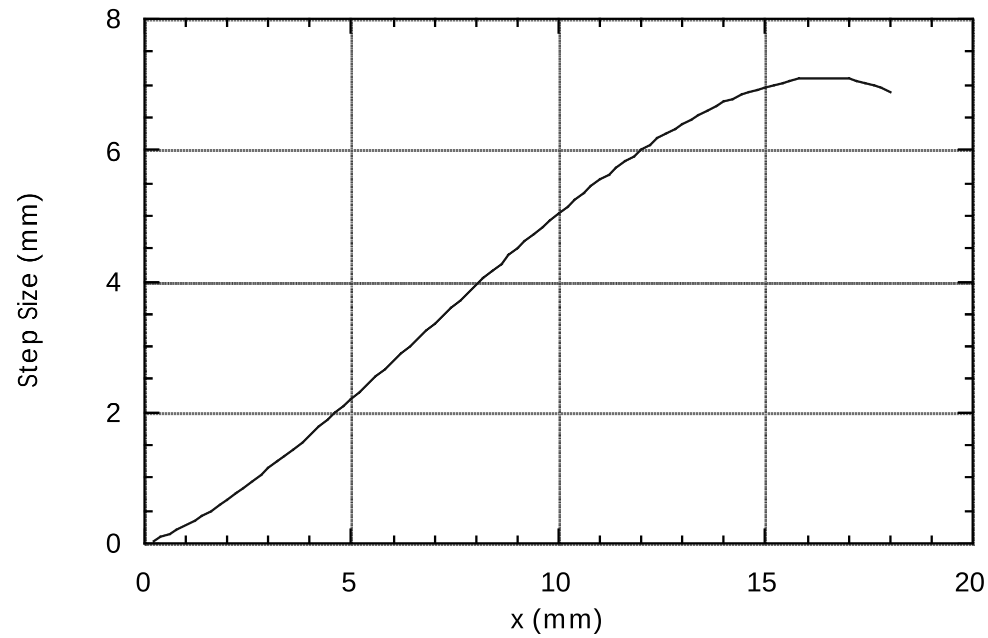


Figure 2.  Step size versus horizontal position.  The step size is the change in position after 2 turns.

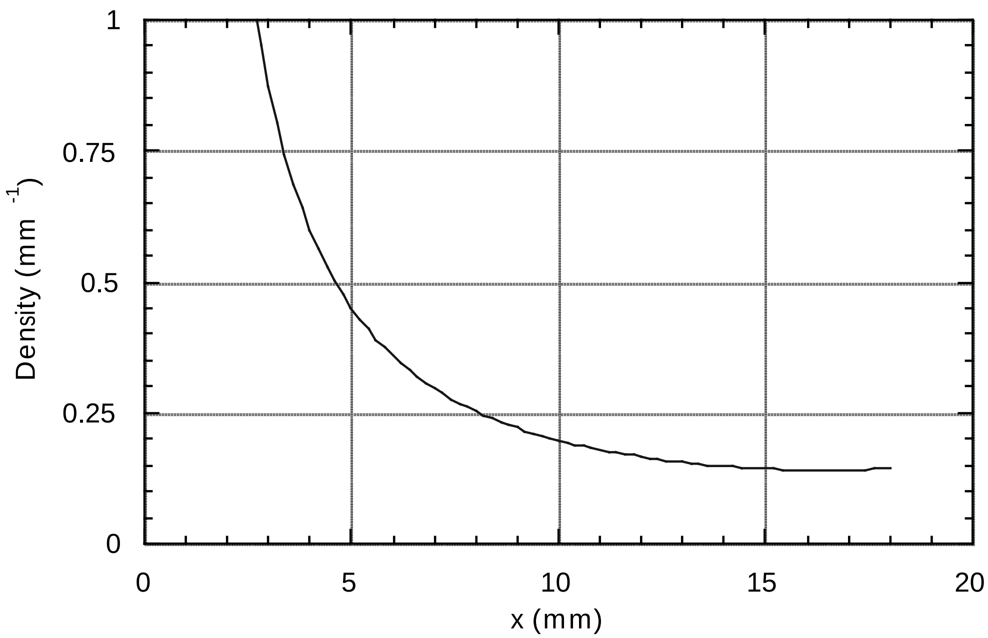


Figure 3.  The relative density versus horizontal position at the septum.  The density in the septum (12 to 18 mm) is approximately uniform.